Advanced Math

9a-4

Proof by Mathematical Induction

Steps for an induction proof.

1) Show that a seed value is true, or show S_1 is true.

2) Assume that S_k is true, and use this formula to derive S_{k+1}.

3) Prove S_{k+1} is also true by showing that $S_{k+1} = S_k + a_{k+1}$.

This sets up a chain reaction. We assume a generic term is true, and from there prove the following term is true. Since we started by showing the seed term is true, our generic model proves the second term is true. Then since the second is true, the generic model proves the third is true. Dominoes!

Prove by mathematical induction.

$a_k = 2k$

$a_{k+1} = 2(k+1)$

5) $\overset{a_1}{2} + \overset{a_2}{4} + \overset{a_3}{6} + \overset{a_4}{8} + \ldots + \overset{a_n}{2n} = n(n+1)$

1) Show that a seed value is true, or show S_1 is true.

since $a_1 = 2$, substitute 1 for n and see if the formula equals two.

$1(1+1) \overset{?}{=} 2$

$2 = 2$

Yes

2) Assume that S_k is true, and use this formula to derive S_{k+1}.

1st put k in for n. $S_k = (k)(k+1)$

Then substitute $k+1$ for n in the formula.

$S_{k+1} = (k+1)[(k+1)+1]$

3) Prove S_{k+1} is also true by showing that $S_{k+1} = S_k + a_{k+1}$.

; $S_{k+1} = S_k + a_{k+1}$

$(k+1)[(k+1)+1] = k(k+1) + 2(k+1)$

$(k+1)(k+2) = (k+1)(k+2)$ $\square$ Cocky box

Since we proved the next after generic term will always be true, and since we proved the first "seed" term is true, the generic proves the second, which proves the third, which proves....

13) $\overset{n=1}{\boxed{1^3}} + \overset{2}{2^3} + \overset{3}{3^3} + \overset{4}{4^3} + \ldots n^3 = \dfrac{n^2(n+1)^2}{4}$

$a_n = n^3$

$a_{k+1} = (k+1)^3$

1) Show S_1 is True

$S_1 \Rightarrow \dfrac{1^2(1+1)^2}{4} \overset{?}{=} 1^3$

$\dfrac{1(2)^2}{4} = 1^3$

$\dfrac{4}{4} = 1$

Yes

2) assume $S_k = \dfrac{k^2(k+1)^2}{4}$ is True

Show $S_{k+1} = S_k + a_{k+1}$

$\dfrac{(k+1)^2((k+1)+1)^2}{4} = \dfrac{k^2(k+1)^2}{4} + \dfrac{4(k+1)^3}{4}$ Common Denom.

FOIL

$\dfrac{(k+1)^2(k+2)^2}{4} = \dfrac{(k+1)^2[k^2+4(k+1)]}{4}$ I took two of these common terms out.

$\dfrac{(k+1)^2(k^2+4k+4)}{4} = \dfrac{(k+1)^2(k^2+4k+4)}{4}$ $\square$

Assignment:
pg. 748
6-18 even.